**FIN500: Problem Set 2**

*Due: Wednesday September 25th before 11:59pm, please submit electronic copies through the assignment function on Canvas*

**1.** Consider a 1-year received-fixed, pay-floating interest rate swap with payments every 3 months based on 3-month SOFR. The fixed rate of the swap is 5.5% percent, the notional principal is USD 100 million, and a floating index value of 5.25 percent is to be used in calculating the first payment. The cash flows of the swap (in millions) are shown in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| Time 0.25 | Time 0.5 | Time 0.75 | Time 1 |
| 0.25(5.5%- 5.25%)  ×100 | 0.25(5.5% - *r*0.25(0.25*,*0.5)  ×100 | 0.25(5.5%- *r*0.5(0.5*,*0.75) ×100 | 0.25(5.5% - *r*0.75(0.75*,*1) ×100 |

The current date is time 0. The swap was transacted 5 minutes ago, and interest rates have changed in the last 5 minutes, so that the current market value of the swap is no longer zero. In particular, the current forward rates are *F*0(0,0.25) = 5.30%, *F*0(0.25,0.5) = 5.35%, *F*0(0.5,0.75) = 5.45%, and *F*0(0.75,1) = 5.55%. These forward rates are expressed on an annual basis, with simple compounding.

*In PS2 Q4, you computed the current market value of the swap as*

*= $98,368.22*

*(I hope this is what you did⎯it is what you were supposed to do.)*

**For this homework assignment**, estimate by how much the value *V* changes if the forward rate *f*0(0.75, 1) changes by one basis point (0.01%), holding the other forward rates fixed at the values *F*0(0,0.25) = 5.30%, *F*0(0.25,0.5) = 5.35%, *F*0(0.5,0.75) = 5.45%,. You may use one of two approaches. Either: (i) differentiate the value *V* with respect to *f*0(0.75, 1), and then evaluate the derivative at *f*0(0.75,1) = 5.55%, and scale the result by 10-4; or (ii) compute a numerical approximation to the derivative by increasing and decreasing *f*0(0.75, 1) by small amounts and recomputing the value of the swap.

*Remark*: The result of your computations will be the “partial” DV01 with respect to the forward rate *f*0(0.75, 1).

**2**. Assume that the expectations hypothesis holds for 3-month interest rates, and that the initial (time 0) forward rates are *r*0(0,0.25) =0.055, *f*0(0.25,0.5) = 0.0525, *f*0(0.5,0.75) = 0.05, and *f*0(0.75,1) = 0.0475. The current date is time 0.

(a) Based on the expectations hypothesis, what are the forecasts for *r*0.25(0.25,0.5), *f*0.25(0.5,0.75), and *f*0.25(0.75,1)?

(b) What are the forecasts for *r*0.5(0.5,0.75) and *f*0.5(0.75,1)? What is the forecast for *r*0.75(0.75,1)?

**3**. (2 points) Luenberger, 4.13. This question is 4.15 in the second edition. A scanned version is below





**4**. Luenberger, 4.4. Please attempt the question from the first edition (scanned below), the question from the second edition has some confusion with the dates. The question says that the current date is November 5, 2011, which suggests that the quotations are from November 5, 2011. This is impossible, because November 5, 2011 is a Saturday. Assume that the quotations are from Friday, November 4, 2011.

*Hin*t: There are 9 dates, 2/15/2011, … , 2/15/2016, and 9 continuously compounded spot rates, that are a function of 5 parameters *a*0, …, *a*4. Thus, the theoretical values of each of the bonds can be expressed as a function of these 5 parameters. But, there are more than 5 bond prices, so it is impossible for all of the quoted bond prices to be consistent with the theoretical values expressed in terms of the 5 parameters *a*0, …, *a*4.

Think of each quoted bond price as being equal to the theoretical bond price, plus an “error” e. That is, letting *Pi* be the quoted price of the *i*th bond and *Vi* be the theoretical value of the *i*th bond, think of each quoted bond price as

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Choose the 5 parameters *a*0, …, *a*4 to minimize the sum of squared residuals  (the sum runs from 1 to 18 because there are 18 bonds).

*Another hint*: This is a non-linear least squares problem. You can compute the estimate using the Excel add-in “Solver.” Or, you may use a statistical software package of your choice.

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**5.** The price of a zero-coupon bond is given by

*V*(*t*0, *T*) = exp[-*r*(*t*0, *T*)(*T* – *t*0)]×100,

where *t*0 is the current date, *T* is the maturity date (so *T* – *t*0 is the time remaining to maturity), and *r*(*t*0, *T*) is the continuously compounded zero-coupon interest rate for the period from *t*0 to *T*.

Please differentiate *V* to obtain a formula for the convexity in terms of the time remaining to maturity *T* – *t*0. Is the convexity increasing or decreasing in *T* – *t*0?

**6**. Consider again the zero-coupon bond in question 7, and suppose that *T* – *t*0 = 10 and *r*(*t*0, *T*) = 0.03 = 3%.

(a) (1 point) Use duration to estimate the change in the bond price if the interest rate increases to 5%.

(b) (1 point) Use duration and convexity to estimate the change in the bond price if the interest rate increases to 5%.

**7**. Suppose that you own a 10-year zero-coupon bond with a face value of $10 million, and a market value of *V*(*t*0, *T*) = exp[-*r*(*t*0, *T*)(*T* – *t*0)]×$10 million = exp[-0.03×10]×$10 million. You want to use 5 and 20-year zero-coupon bonds to form a portfolio that has Fisher-Weil duration and convexity both equal to zero. The continuously compounded 5 and 20-year zero-coupon interest rates are 0.015 = 1.5% and 0.035 = 3.5%, respectively. What quantities of the 5 and 20-year zero-coupon bonds should you buy or sell?

*Hint*. You continue to hold the 10-year bond.

*Another hint*. You will have an additional equation that requires the portfolio convexity to be zero. You can set this problem up with two equations requiring that both duration and convexity be zero, and not worry about matching the values. Or, you can set it up with three equations that require duration convexity both equal zero, and the value of the portfolio match the initial value. If you take the latter approach and require that the value of the portfolio match the initial value, then you should also introduce a “cash” account that has duration and convexity of zero.